MENIIT

NEET | IIT-JEE | FOUNDATION

Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 26-02-2021 (MORNING SHIFT) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

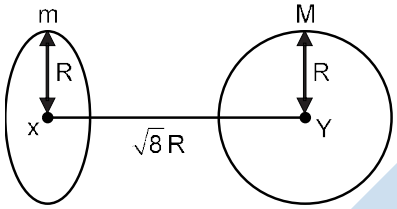
QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.



(1) $\frac{\sqrt{8}}{9}\frac{GmM}{R}$ (2) $\frac{2\sqrt{2}}{3}\frac{GMm}{R^2}$ (3) $\frac{1}{3\sqrt{8}}\frac{GMm}{R^2}$ (4*) $\frac{\sqrt{8}}{27}\frac{GmM}{R^2}$

Ans. (4)

Sol. Gravitational field of ring

$$\frac{Gmx}{(R^2 \quad x^2)^{3/2}}$$

Force between sphere & ring

$$\frac{GmM\ \sqrt{8}R}{(R^2 \quad 8R^2)^{3/2}} \qquad \frac{GmM}{R^2}\frac{\sqrt{8}}{27}$$

2. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ time the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

(1) $\frac{15}{11}$ (2) $\frac{111}{80}$ (3) $\frac{29}{15}$ (4) $\frac{15}{4}$

Ans. (2)

Sol. (Bonus)

When connected in parallel

$C_{eq} = C_1 + C_2$

When in series

$$C'_{eq} \quad \frac{C_1C_2}{C_1 \quad C_2} \qquad C_1 \quad C_2 \quad \frac{15}{4}\frac{C_1C_2}{C_1 \quad C_2}$$

$4(C_1 + C_2)^2 = 15\ C_1C_2$ $4C_1^2\ \ 4C_2^2\ \ 7C_1C_2\ \ 0$

Dividing by C_1^2

$4\ \frac{C_2}{C_1}^{\ 2}\ \ \frac{7C_2}{C_1}\ \ 4\ \ 0$

Let $\frac{C_2}{C_1}\ \ x$

$4x^2 - 7x + 4 = 0$

$b^2 - 4ac = 49 - 64 < 0$

No. solution exits

3. In typical combustion engine the work done by a gas molecule is given $W\ \ ^2\ e^{\frac{x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be :

(1) $[MLT^{-2}]$ (2) $[M^0LT^0]$ (3) $[M^2LT^{-2}]$ (4) $[MLT^{-1}]$

Ans. (2)

Sol. kT has dimension of energy

$\frac{x^2}{kT}$ is dimensionless

$[\beta][L^2] = [ML^2T^{-2}]$

$[\beta] = [MT^{-2}]$

$\alpha^2\beta$ has dimensions of work

$[\alpha^2][MT^{-2}] = [ML^2T^{-2}]$

$[\alpha] = [M^0LT^0]$

4. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member fo the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is :

(1) 1 : 9 (2) 7 : 108 (3) 7 : 135 (4) 1 : 3

Ans. (3)

Sol. $\frac{1}{_1}\ \ R\ \frac{1}{1^2}\ \ \frac{1}{4^2}$

$\frac{1}{_2}\ \ R\ \frac{1}{3^2}\ \ \frac{1}{4^2}$

$\frac{_1}{_2}\ \ \frac{\frac{1}{9}\ \ \frac{1}{16}}{1\ \ \frac{1}{16}}\ \ \frac{7}{9\ \ 15}$ $\frac{_1}{_2}\ \ \frac{7}{135}$

5. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value |f| = 40 cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information :

(1) Image is real, same side of concave mirror.

(2) Image is virtual, opposite side of concave mirror.

(3) Image is real, same side of convex mirror.

(4) Image is virtual, opposite side of convex mirror.

Ans. (4)

Sol. Since orientation is same image is virtual. Since image is smaller the mirror has to be convex.

6. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

(1) $\frac{2\ R}{g}$ (2) $\frac{g}{2\ R}$ (3) $\frac{1}{2}\sqrt{\frac{g}{R}}$ (4) $2\sqrt{\frac{R}{g}}$

Ans. (4)

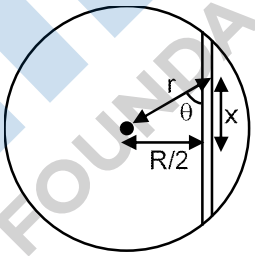
Sol. Force along the tunnel

$$F \quad \frac{GMmr}{R^3} \cos$$

$$F \quad \frac{gm}{R} x \ \frac{GM}{R^2} \quad g, r\cos \quad x$$

$$a \quad \frac{g}{R} x$$

$$^2 \quad \frac{g}{R} \qquad T \quad 2\sqrt{\frac{R}{g}}$$



7. An alternating current is given by the equation $i = i_1 \sin\omega t + i_2 \cos\omega t$. The rms current will be :

(1) $\frac{1}{\sqrt{2}} i_1^2 \quad i_2^2 \ ^{\frac{1}{2}}$ (2) $\frac{1}{\sqrt{2}}(i_1 \quad i_2)^2$ (3) $\frac{1}{2} i_1^2 \quad i_2^2 \ ^{\frac{1}{2}}$ (4) $\frac{1}{\sqrt{2}}(i_1 \quad i_2)$

Ans. (1)

Sol. $i = i_1 \sin\omega t + i_2 \sin(\omega t + 90)$

$$i \quad \sqrt{i_1^2 \quad i_2^2} \sin(\quad t \quad)$$

$$i_{rms} \quad \frac{i_0}{\sqrt{2}} \quad \frac{\sqrt{i_1^2 \quad i_2^2}}{\sqrt{2}}$$

8. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be :

(1) $\frac{K}{P}$ (2) $\frac{P}{K}$ (3) $\frac{K}{P}$ (4) $\frac{PK}{\quad}$

Ans. (2)

Sol. $\frac{M}{V}$

$\frac{d}{} \quad \frac{dV}{V}$

$k \quad \frac{P}{\frac{dV}{V}}$

$\frac{dV}{V} \quad \frac{P}{k}$

$\frac{d}{} \quad \frac{P}{k} \quad d \quad \frac{P}{k}$

9. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. I*ts time period of revolution will be given by :

(1) $T \propto R^2$ (2) $T \quad R^{\frac{3}{2}}$ (3) $T \quad R^{\frac{5}{2}}$ (4) $T \quad R^{\frac{4}{3}}$

Ans. (1)

Sol. $F \quad \frac{1}{R^3}$

$\frac{K}{R^3} \quad m \quad ^2R$

$^2 \quad \frac{K}{m} \quad \frac{1}{R^4}$

$\frac{2}{T} \quad ^2 \quad \frac{K}{m} \quad \frac{1}{R^4}$

$T^2 \propto R^4$

$T \propto R^2$

10. A planet revolving in elliptical orbit has :

(A) a constant velocity of revolution.

(B) has the least velocity when it is nearest to the sun.

(C) it areal velocity is directly proportional to its velocity.

(D) areal velocity is inversely proportional to its velocity.

(E) to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below :

(1) A only (2) D only (3) C only (4) E only

Ans. (4)

Sol. As per Keppler's 2nd law, Areal velocity is constant.

11. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m << M, body 'Q' will have a maximum speed equal to '2u' after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is not correct but R is correct.

(2) Both A and R are correct but R is NOT the correct explanation of A.

(3) Both A and R are correct and R is the correct explanation of A.

(4) A is correct but R is not correct.

Ans. (3)

Sol. For e = 1 & second body at reast

$$V_2 \quad \frac{2m_1u_1}{m_1 \quad m_2} \quad \frac{2u(M)}{M \quad m} \simeq 2u$$

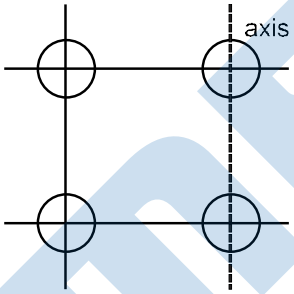
Since M >> m

12. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is :

(1) $\frac{4}{5}ma^2 \quad 2mb^2$ (2) $\frac{8}{5}ma^2 \quad mb^2$ (3) $\frac{8}{5}ma^2 \quad 2mb^2$ (4) $\frac{4}{5}ma^2$

Ans. (3)

Sol.



$$I \quad 2 \quad \frac{2}{5}ma^2 \quad 2 \quad \frac{2}{5}ma^2 \quad mb^2$$

$$I \quad \frac{8}{5}ma^2 \quad 2mb^2$$

13. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be:

(1) 0.25 mm (2) 0.50 mm (3) 0.75 mm (4) 1 mm

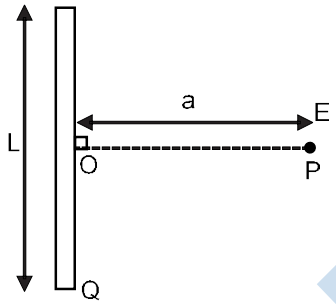
Ans. (1)

Sol. $\frac{D}{d} \quad \frac{500 \quad 10^{9} \quad 1}{2 \quad 10^{3}}$

$\frac{5}{2}$ 10^{-4}m 2.5 10^{-1}mm

b = 0.25 mm

14. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is a $\frac{\sqrt{3}}{2}L$.



(1) $\frac{\sqrt{3}Q}{4\ _0L^2}$ (2) $\frac{Q}{3\ _0L^2}$ (3) $\frac{Q}{2\sqrt{3}\ _0L^2}$ (4) $\frac{Q}{4\ _0L^2}$

Ans. (3)

Sol. $E \frac{k}{a}(\sin\ _1 \sin\ _2)$

$E \frac{1}{4\ _0} \frac{Q}{L} \frac{1}{\frac{\sqrt{3}L}{2}} (2\sin\ )$

$\tan \frac{L/2}{\frac{\sqrt{3}L}{2}} \frac{1}{\sqrt{3}}$

$\sin \frac{1}{2}$

$E \frac{1}{4\ _0} \frac{2Q}{\sqrt{3}L^2} 2 \frac{1}{2}$

$E \frac{Q}{2\sqrt{3}\ _0L^2}$

15. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor :

(1) $\frac{1}{2},\sqrt{2}$ (2) $\frac{1}{4},\sqrt{2}$ (3) $\frac{1}{4},2\sqrt{2}$ (4) $\frac{1}{2},2\sqrt{2}$

Ans. (1)

Sol. $\frac{1}{k_{eq}} \frac{1}{k_1} \frac{1}{k_2}$

$$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{k} \quad k_{eq} = \frac{k}{2}$$

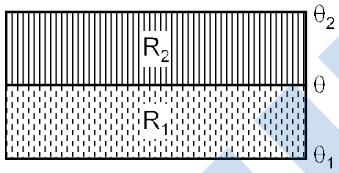
$$k' = \frac{k}{2}$$

$$T = 2\sqrt{\frac{M}{k}} \qquad T' = 2\sqrt{\frac{M}{k'}}$$

$$T' = 2\sqrt{\frac{M}{k}} \sqrt{2}$$

$$T' = \sqrt{2}T$$

16. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by :

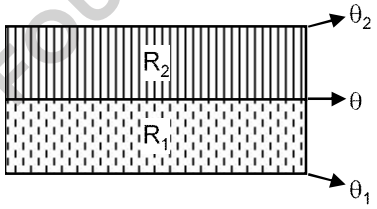


(1) $\frac{{}_2R_2 \quad {}_1R_1}{R_2 \quad R_1}$ (2) $\frac{{}_1R_2 \quad {}_2R_1}{R_2 \quad R_1}$ (3) $\frac{{}_1R_2 \quad {}_2R_1}{R_1 \quad R_2}$ (4) $\frac{{}_1R_1 \quad {}_2R_2}{R_1 \quad R_2}$

Ans. (3)

Sol. Heat flow rate will be same through both

$$\frac{{}^{1}}{R_1} \quad \frac{{}^{2}}{R_2}$$



$$R_2\theta_1 - R_2\theta = R_1\theta - R_1\theta_2$$

$$\frac{R_2 \ {}_1 \quad R_1 \ {}_2}{R_1 \quad R_2}$$

17. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : An electron microscope can achieve better resolving power than an optical microscope.

Reason R : The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light. In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false.

(2) Both A and R are true and R is the correct explanation of A.

(3) Both A and R are true but R is NOT the correct explanation of A.

(4) A is false but R is true.

Ans. (2)

Sol. Resolving power $\frac{1}{}$

Since wavelength of electron is much less than visible light, its resolving power will be much more.

18. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.

[$h = 6.63 \times 10^{-34}$ Js and $c = 3 \times 10^{8}$ ms^{-1}]

(1) 1046 nm and red colour (2) 654 nm and orange colour

(3) 1046 nm and blue colour (4) 654 nm and red colour

Ans. (4)

Sol. $\frac{hc}{E} \quad \frac{6.6 \quad 10^{34} \quad 3 \quad 10^{8}}{1.9 \quad 1.6 \quad 10^{19}} \quad 6.54 \quad 10^{7} = 644$ nm

Red color

19. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be:

(1) $\frac{2T}{J} \frac{1}{r} \frac{1}{R}$ (2) $\frac{2T}{rJ}$ (3) $\frac{3T}{rJ}$ (4) $\frac{3T}{rJ} \frac{1}{r} \frac{1}{R}$

Ans. (4)

Sol. $n \frac{4}{3} r^3 \frac{4}{3} R^3$

$\therefore \quad n^{1/3} r = R$

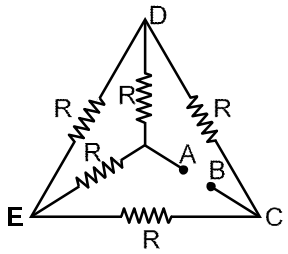
$\therefore$ Total change in surface energy = $(n(4\pi r^2) - 4\pi R^2)T$

$\Rightarrow \quad 4\pi T (nr^2 - R^2)$

$\therefore$ Heat energy $\frac{4 \quad T(nr^2 \quad R^2)}{J \quad \frac{4}{3} \quad R^3} \quad \frac{3T}{J} \frac{nr^2}{R^3} \frac{1}{R}$

Put $nr^3 = R^3$ $\frac{3T}{J} \frac{1}{r} \frac{1}{R}$

20. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is :

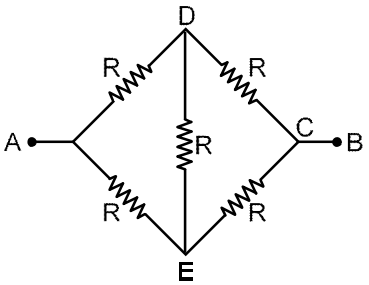


(1) 2R (2) $\frac{R}{2}$

(3) $\frac{3R}{2}$ (4) R

Ans. (4)

Sol. This diagram can be drawn like



It is a wheat stone bridge

$R_{eq} \quad \frac{2R \quad 2R}{2R \quad 2R} \quad R$

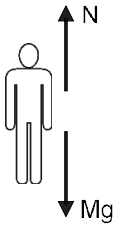
Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be_ N. [g = 10 m/s^2]

Ans. (492)

Sol.

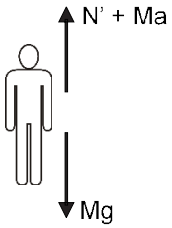


When lift is at rest

N = mg

$\Rightarrow$ 60 × 10 = 600 N

When lift moves with downward acceleration. In frame of lift pseudo force will be in upward direction.



N' = M (g – a)

$\Rightarrow$ 60 (10 – 1.8)

N ' $\Rightarrow$ 492 N

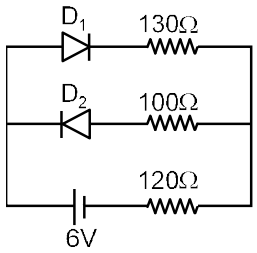
2. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is _____J.

Ans. (300)

Sol. Work done by battery = Q (ΔV)

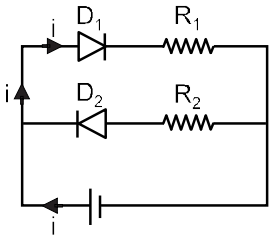
$\Rightarrow$ 20 × 15 = 300 J

3. The circuit contains two diodes each with a forward resistance of 50 W and with infinite reverse resistance. If the battery voltage is 6 V, the current through the 120 W resistance is_ mA.



Ans. (20)

Sol.



In this circuit D_1 will be forward bias and D_2 will be revers bias.

$\therefore$ These will be no current through D_2 and R_2 apply KVL in circuit we bet

$+ 6 - 50i - 130i - 120i = 0$

$i \quad \frac{6}{200} A \quad \frac{6}{300} \quad 1000mA$

$\Rightarrow 20$ mA

4. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is_. (Rounded-off to the nearest integer)

[Take $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}\ m^{-2}$, $c = 3 \times 10^8\ ms^{-1}$]

Ans. (137)

Sol. $I_{avg} \quad \frac{1}{2} \ {}_0E_0^2C$

$\frac{1.25}{100} \quad \frac{1000}{4 \ (2)^2} \quad \frac{1}{2} \quad 8.85 \quad 10^{\ 12} \quad 3 \quad 10^8 \quad E_0^2$

$E_0^2 \quad 187.4$

$\therefore E_0 = 13.689$ V/m $= 136.89 \times 10^{-1}$ V/m

$\therefore x = 136.89$

Rounding off to nearest integer

X = 137

5. A boy pushes a box of mass 2 kg with a force $\vec{F} \quad 20\hat{i} \quad 10\hat{j}$ N on a frictionless surface. If the box was initially at rest, then _____ m is displacement along the x-axis after 10 s.

Ans. (500)

Sol. $\vec{F} \quad 20\hat{i} \quad 10\hat{j}$

$\vec{a} \quad \frac{\vec{F}}{m} \quad \frac{20\hat{i} \quad 10\hat{j}}{2} \quad 10\hat{i} \quad 5\hat{j}$

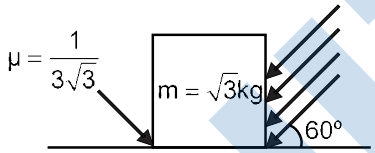
$\vec{s} \quad \frac{1}{2}\vec{a}t^2 \quad \frac{1}{2}\,10\hat{i} \quad 5\hat{j} \quad (10)^2$

$50\ 10\hat{i} \quad 5\hat{j}\ m$

$\therefore$ Displacement along x-axis

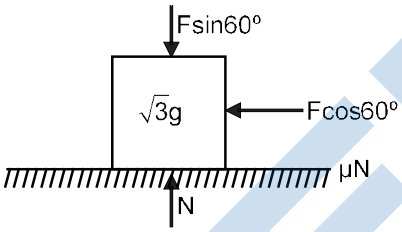
6. As shown in the figure, a block of mass $\sqrt{3}$kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle 60º with horizontal such that it does not move, will be 3x. The value of x will be :

[g = 10 m/s^2 ; $\sin 60^\circ \quad \frac{\sqrt{3}}{2}; \cos 60^\circ \quad \frac{1}{2}$]



Ans. (10)

Sol.



$F\cos 60^\circ \quad \mu N \text{ or } \frac{F}{2} \quad \frac{1}{3\sqrt{3}}N$ ....(1)

& $N \quad \sin 60^\circ \quad \sqrt{3}g$ ....(2)

From equation (1) & (2)

$\frac{F}{2} \quad \frac{1}{3\sqrt{3}} \quad \frac{F\sqrt{3}}{2} \quad \sqrt{3}g$

$\Rightarrow$ F = g = 10 Newton = 3x

So $x \quad \frac{10}{3} \quad 3.33$

7. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is $x \times 10^{-1}$ atm. Value of x is_.

Ans. **(25)**

Sol. Let common equilibrium pressure of mixture is P atmp. Then

$$U_1 + U_2 = U_{mixture}$$

$$\frac{f}{2}P_1V_1 \quad \frac{f}{2}P_2V_2 \quad \frac{f}{2}P(V_1 \quad V_2)$$

$$\frac{f}{2}(2)(4.5) \quad \frac{f}{2}(3)(5.5) \quad \frac{f}{2}P(4.5 \quad 5.5)$$

$\Rightarrow P = 2.55 = x \times 10^{-1}$ atmp

So $x = 25.5 \approx 25$

8. the mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y = -0.21 \sin(x + 30t)$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is : (Round-off to the nearest integer)

Ans. (12)

Sol. $\mu = 0.135$ gm/cm $= 0.0135$ kg/m

$y = -0.21 \sin(x + 30t)$ (x in meter & t in sec)

$$v \quad \frac{}{k} \quad \frac{30}{1} \quad 30\,m/s$$

$$v \quad \sqrt{\frac{T}{\mu}} \quad T \quad v^2\mu \quad (30)^2(0.0135) \quad = 12.15$$

$= x \times 10^{-2}$ N

$\Rightarrow x = 1215$

9. In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be_____.

Ans. (400)

Sol. $$Q \quad \frac{X_L}{R} \quad \frac{L}{R} \quad \frac{1}{\sqrt{LC}}\frac{L}{R} \quad \frac{\sqrt{L}}{R\sqrt{C}}$$

$$Q' \quad \frac{\sqrt{2L}}{\frac{R}{2}\sqrt{C}} \quad 2\sqrt{2}Q \quad 2\sqrt{2}(100) \quad = 282.84$$

10. The maximum and minimum amplitude of an amplitude modulated wave is 16V and 8V respectively. The modulation index for this amplitude modulated wave is $x \times 10^{-2}$. The value of x is_____.

Ans. (33)

Sol. Modulation index $\frac{A_{max} \quad A_{min}}{A_{max} \quad A_{min}}$

$$\frac{16 \quad 8}{16 \quad 8} \quad \frac{8}{24} \quad \frac{1}{3} \quad 0.33$$

$x + 10^{-2} = 0.33$

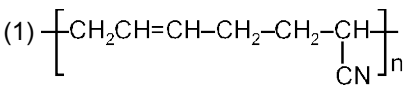
x = 33

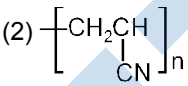
PART B : CHEMISTRY

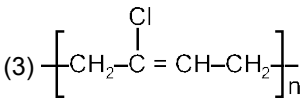
Single Choice Type

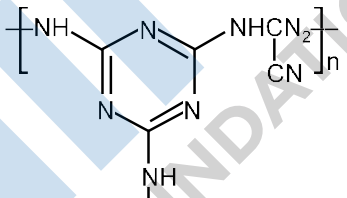
This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The structure of Neoprene is :

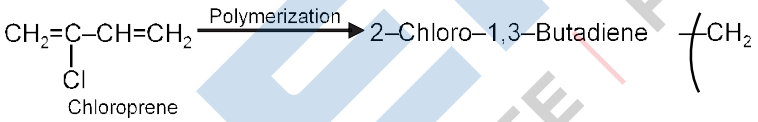
(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. 

2. Find A, B and C in the following reactions :

$NH_3 + A + CO_2 \rightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + B \rightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$

(1) A – O_2 ; B – CO_2 ; C – Na_2CO_3

(2) A – H_2O ; B – O_2 ; C – Na_2CO_3

(3) A – H_2O ; B – O_2 ; C – $NaHCO_3$

(4) A – H_2O ; B – CO_2 ; C – $NaHCO_3$

Ans. (4)

Sol. $2NH_3 \quad \underset{(A)}{H_2O} \quad CO_2 \quad (NH_4)_2CO_3$

$(NH_4)_2CO_3 \quad H_2O \quad \underset{(B)}{CO_2} \quad 2NH_4HCO_3$

$NH_4HCO_3 \quad NaCl \quad \underset{(C)}{NaHCO_3} \quad NH_4Cl$

3. The presence of ozone in troposphere

(1) Protects us from the UV radiation (2) Protects us from the X-ray radiation

(3) Protects us from greenhouse effect (4) generates photochemical smog

Ans. (4)

Sol. The presence of ozone in troposphere generates photochemical smog.

4. Match List -I with List - II

List – I	**List - II**
Electronic configuration of elements	Δ_i in kJ mol^{-1}
(a) $1s^2 2s^2$	(i) 801
(b) $1s^2 2s^2 2p^4$	(ii) 899
(c) $1s^2 2s^2 2p^3$	(iii) 1314
(d) $1s^2 2s^2 2p^1$	(iv) 1402

Choose the most appropriate answer from the options given below :

(1) (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)

(2) (a) → (i), (b) → (iv), (c) → (iii), (d) → (ii)

(3) (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)

(4) (a) → (iv), (b) → (i), (c) → (ii), (d) → (iii)

Ans. (1)

Sol. (a) $1s^2\ 2s^2 \rightarrow$ Be

(b) $1s^2\ 2s^2\ 2p^4 \rightarrow$ O

(c) $1s^2\ 2s^2\ 2p^3 \rightarrow$ N

(d) $1s^2\ 2s^2\ 2p^1 \rightarrow$ B

The ionization enthalpy order is B < Be < O < N

Be has more IE compared to B due to extra stability & N has more IE compared to oxygen due to extra stability

Hence, N → 1402 kJ/mol

O → 1314 kJ/mol

B → 801 kJ/mol

Be → 899 kJ/mol

5. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation.

Reason R : Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) A is false but R is true

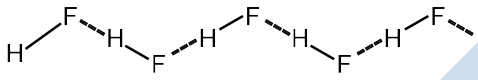
(2) Both A and R are true and R is the correct explanation of A

(3) A is true R is false

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. (1)

Sol. Assertion is incorrect since in hydrogen bonding, Dipole–dipole interactions are noncovalent but ion-dipole interaction can also result in H-bond formation. Reason is correct since F is most electronegative element & structure is



Symmetrical H-bonds are present

6. Statements about heavy water are given below.

(A) Heavy water is used in exchange reactions for the study of reaction mechanisms.

(B) Heavy water is prepared by exhaustive electrolysis of water

(C) Heavy water has higher boiling point than ordinary water.

(D) Viscosity of H_2O is greater than D_2O

(1) A, B and C only (2) A and B only (3) A and D only (4) A and C only

Ans. (1)

Sol. Heavy water is used in exchange reactions for study of reaction mechanisms

Heavy water is prepared by exhaustive electrolysis of water.

B.P. of D_2O = 374.4 K

B.P. of H_2O = 373 K

Viscosity of H_2O = 0.89 centipoise

Viscosity of D_2O = 1.107 centipoise

7. The orbital having two radial as well as two angular nodes is :

(1) 3p (2) 4f (3) 4d (4) 5d

Ans. (4)

Sol. $n - \ell - 1 = 2$

$\ell = 2$

$n - 2 - 1 = 2$

$n = 5$

8. Match List -I with List - II

List – I	**List – II**
(Ore)	**(Element Present)**

(a) Kernite	(i) Tin
(b) Cassiterite	(ii) Boron
(c) Calamine	(iii) Fluorine
(d) Cryolite	(iv) Zinc

Choose the most appropriate answer from the options given below.

(1) (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)

(2) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

(3) (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)

(4) (a) → (iii), (b) → (i), (c) → (ii), (d) → (iv)

Ans. (2)

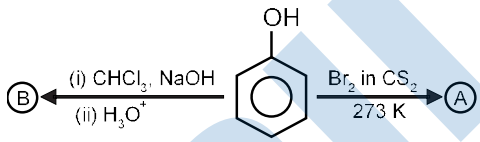
Sol. Kernite = $Na_2B4O_7.4H_2O$

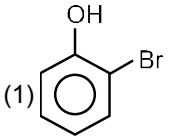
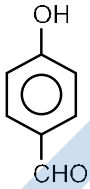
Cassiterite = SnO_2

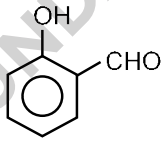
Calamine = $ZnCO_3$

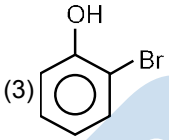
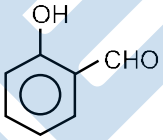
Cryolite = Na_3AlF_6

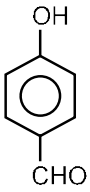
9. Identify the major products A and B respectively in the following reactions of phenol.



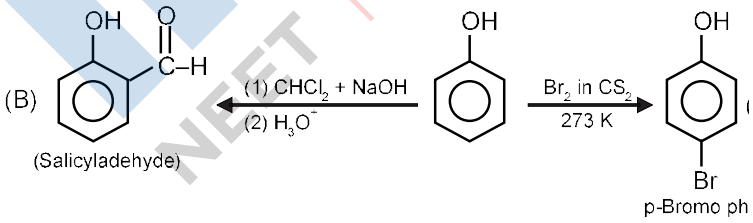
(1)  and 

(2)  and 

(3)  and 

(4)  and 

Ans. (2)

Sol. 

10. Given below are two statements :

Statement I : A mixture of chloroform and aniline can be separated by simple distillation.

Statement II : When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point. In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement-I is false but Statement II is true (2) Both Statement-I and Statement II are false

(3) Statement-I is true but Statement II is false (4) Both Statement-I and Statement II are true

Ans. (4)

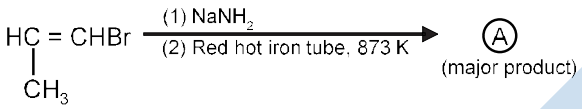
Sol. **Statement 1 :** B.P. of chloroform = 334 K

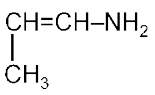
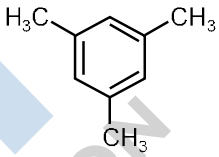
B.P. of aniline = 457 K

thus can be separated of simple distillation.

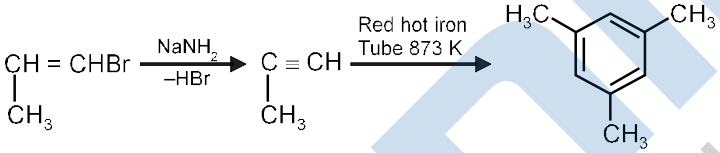
Statement 2 : Mixture of aniline and water separated by simple distillation.

11. For the given reaction :



(1) $CH_3CH_2CH_2NH_2$ (2)  (3)  (4) 

Ans. (4)

Sol. 

12. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are :

(1) X = SO_2, Y = Cr_2O_3 (2) X = SO_3, Y = Cr_2O_3

(3) X = SO_2, Y = $Cr_2(SO_4)_3$ (4) X = SO_3, Y = $Cr_2(SO_4)_3$

Ans. (3)

Sol. $SO_2 + \text{dil } H_2SO_4 \longrightarrow SO_3(g)$

$SO_3 \quad K_2Cr_2O_7 \xrightarrow[H_2SO_4]{\text{dil.}} Cr_2(SO_4)_3$

13. Which of the following is 'a' FALSE statement?

(1) Carius tube is used in the estimation of sulphur in an organic compound

(2) Carius method is used for the estimation of nitrogen in an organic compound

(3) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture.

(4) Kjeldahl's method is used for the estimation of nitrogen in an organic compound

Ans. (2)

Sol. Carius method is used in the estimation of halogen in organic compounds.

14. Which of the following vitamin is helpful in delaying the blood clotting :

(1) Vitamin C (2) Vitamin B (3) Vitamin E (4) Vitamin K

Ans. (4)

Sol. Vitamin helpful in delaying the blood clotting is Vitamin K

15. $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373\ K]{\text{Hydrolysis}} \underset{(C_4H_8O)}{B}$

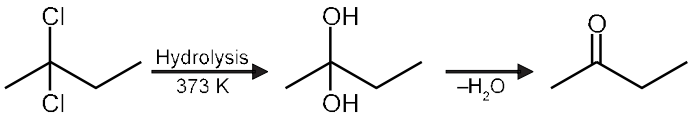
B reacts with Hydroxyl amine but does not give Tollen's test. Identify A and B

(1) 1,1-Dichlorobutane and 2-Butanone (2) 2,2-Dichlorobutane and Butanal

(3) 1,1-Dichlorobutane and Butanal (4) 2,2-Dichlorobutane and 2-butan-one

Ans. (4)

Sol.



$(C_4H_8Cl_2)$ (A) (C_4H_8O) (B)

16. Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is :

(1) PbO_2 (2) PbO (3) $PbSO_4$ (4) Pb_3O_4

Ans. (1)

Sol. PbO_2 is amphoteric and strong oxidizing agent and also a component of lead storage batteries.

17. Which one of the following lanthanoids does not form MO_2? [M is lanthanoid metal]

(1) Pr (2) Dy (3) Nd (4) Yb

Ans. (4)

Sol. Yb is the only element that do not form MO_2 type oxide

18. Given below are two statements :

Statement I : o-Nitrophenol is steam volatile due to intramolecular hydrogen bonding.

Statement II : o-Nitrophenol has high melting due to hydrogen bonding. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is false but Statement II is true

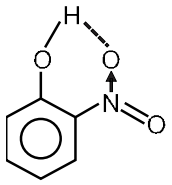
(2) Both statement I and statement II are true

(3) Both statement I and statement II are false

(4) Statement I is true but statement II is false

Ans. (4)

Sol.

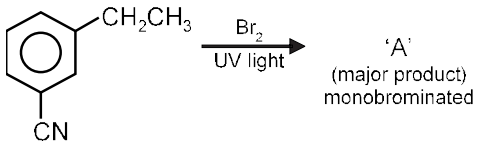


Intramolecular H-bonding

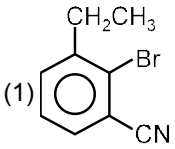
thus it is more volatile due to intramolecular H-bonding.

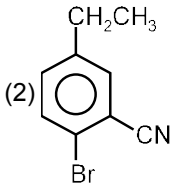
Melting point depends on packing efficiency not on H-bonding thus statement II is false

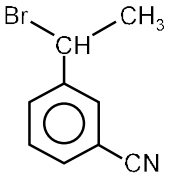
19. For the given reaction :

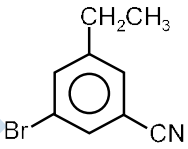


What is 'A' ?

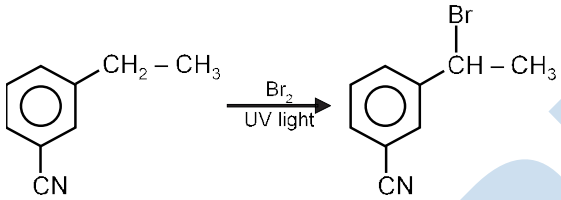
(1) 

(2) 

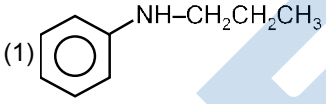
(3) 

(4) 

Ans. (3)

Sol. 

20. An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is :

(1) 

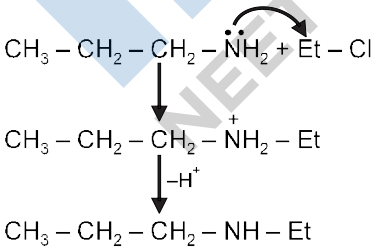
(2) $CH_3CH_2NH_2$

(3) $CH_3CH_2CH_2NHCH_3$

(4) $CH_3CH_2CH_2\overset{H}{N}\ CH_3CH_3$

Ans. (4)

Sol. It has to be 2° amine because on reaction with benzene sulphonylchloride it gives water in soluble product. As it is formed by ammonolysis of ethylchloride, so it has to be R–NH–Et type.



Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. For a chemical reaction A + B $\rightleftharpoons$ C + D ($\Delta_f H^\circ$ = 80 kJ mol^{-1}) the entropy change $\Delta_r S^\circ$ depends on the temperature T (in K) as ($\Delta_r S^\circ$ = 2T (J K^{-1} mol^{-1})).

Minimum temperature at which it will become spontaneous is _____ K. (Integer)

Ans. (200)

Sol. $\Delta G^\circ = \Delta H^\circ - T \times \Delta S^\circ$

$\Delta G^\circ = \Delta H^\circ - T \times (2T)$

T = 200 K

2. The number of significant figures in 50000.020 × 10^{-3} is _____.

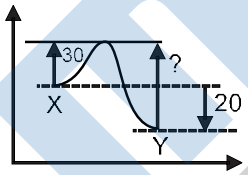
Ans. (7)

Sol. 50000.020 × 10^{-3}

3. An exothermic reaction X $\rightarrow$ Y has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is –20 kJ, then the activation energy for the reverse reaction in kJ is _____.(Integer answer)

Ans. (50)

Sol. X $\longrightarrow$ Y



4. Consider the following reaction

MnO_4 8H 5e Mn^2 $4H_2O, E^\circ$ 1.51V

The quantity of electricity required in Faraday to reduce five moles of MnO_4 is____.

Ans. (25)

5. A certain gas obeys $P(V_m - b) = RT$. The value of $\left(\frac{Z}{dP}\right)_T$ is $\frac{xb}{RT}$. The value of x is _____.

(Z : compressibility factor)

Ans. (1)

Sol. $Z \quad 1 \quad \frac{Pb}{RT}$

$$\frac{Z}{dP}_{T} \quad 0 \quad \frac{b}{RT} \quad 1$$

6. A homogeneous ideal gaseous reaction $AB_2(g) \rightleftharpoons A(g) + 2B(g)$ is carried out in a 25 litre flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm. The value of K_P is $x \times 10^{-2}$. The value of x is_____.

Ans. (74)

Sol. $AB_2 = A + 2B$

$1 \quad - \quad -$

$1-\alpha \quad \alpha \quad 2\alpha$

$= 0.535 \quad 0.465 \quad 0.93$

$1.9 \times 25 = n_T \times 0.08206 \times 300$

$n_T = 1.93 = 1 + 2\alpha$

$\alpha = 0.465$

$$Kp \quad \frac{\frac{0.465}{1.93} \quad 19 \quad \frac{0.93}{1.93} \quad 1.9}{\frac{0.535}{1.93} \quad 1.9}^{2}$$

$\simeq 73 \times 10^{-2}$ atm^2

7. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is _____.

Ans. (6)

Sol. $Cr_2O_7^{2} \quad OH \quad CrO_4^{2}$

Oxidation state of Cr in CrO_4^{2} is +6

8. 224 mL of $SO_2(g)$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute)

$P_{(H_2O)}$ 24 mm of Hg is $x \times 10^{-2}$ mm of Hg, the value of x is _____.

Ans. (12)

Sol. $SO_2 + NaOH \rightarrow NaHSO_3$

$9.2 \quad 10 \quad -$

$- \quad 0.8 \quad 9.2$

$\Delta P = P^{o} \; . \; X_{solute}$

$$24 \quad \frac{(1.6 \quad 18.4)}{2020}$$

$= 0.2376 = 23.76 \times 10^{-2}$

9. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is_____.

[R = 0.0821 L atm K^{-1} mol^{-1}]

Ans. (2)

Sol. $V \quad \frac{\frac{3.12}{32} \quad 0.0821 \quad 300}{1} \quad 2.40\ell$

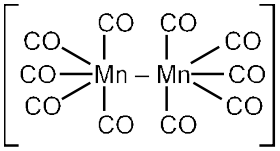
$\because$ 1.2 gm adsorbs 2.40 ℓ

$\therefore$ 1 gm adsorbs 2 ℓ

10. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is _____.

Ans. (0)

Sol. $Mn_2(CO)_{10}$ structure is



PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\ \ \vec{a}\ \ \vec{a}\ \ \vec{a}\ \ \vec{b}$ is equal to :

(1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4\ \vec{b}$ (3) $\vec{a}\ \ \vec{b}$ (4) $|\vec{a}|^4\ \vec{b}$

Ans. (4)

Sol. $\vec{a}\ \vec{b}\ \ 0$

$\vec{a}\ \ \vec{a}\ \ \vec{b}\ \ \ \vec{a}\ \vec{b}\ \vec{a}\ \ \ \vec{a}\ \vec{a}\ \vec{b}\ \ \ \ |\vec{a}^2|\vec{b}$

Now $\vec{a}\ \ \vec{a}\ \ \ |\vec{a}|^2\ \vec{b}$

$|\vec{a}|^2\ \ \vec{a}\ \ \vec{a}\ \ \vec{b}$

$|\vec{a}|^2\ \ \ |\vec{a}|^2\ \vec{b}\ \ \ |\vec{a}|^4\ \vec{b}$

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{14}}$

Ans. (1)

Sol. Let the coin be tossed n-times

P(H) P(T) $\frac{1}{2}$

P(7 heads) ${}^nC_7\ \frac{1}{2}^{n\ 7}\ \frac{1}{2}^{7}\ \ \frac{{}^nC_7}{2^n}$

P(9 heads) ${}^nC_9\ \frac{1}{2}^{n\ 9}\ \frac{1}{2}^{9}\ \ \frac{{}^nC_9}{2^n}$

P(7 heads) = P(9 heads)

${}^nC_7 = {}^nC_9 \Rightarrow n = 16$

P(2 heads) $^{16}C_2 \left(\frac{1}{2}\right)^{14} \left(\frac{1}{2}\right)^{2} \quad \frac{15 \quad 8}{2^{16}}$

P(2 heads) $\frac{15}{2^{13}}$

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

(1) 4 (2) 1 (3) 6 (4) 12

Ans. (1)

Sol. $A \quad \begin{matrix} a & b \\ b & c \end{matrix}, \quad a,b,c \quad I$

$A^2 \quad \begin{matrix} a & b \\ b & c \end{matrix} \quad \begin{matrix} a & b \\ b & c \end{matrix} \quad \begin{matrix} a^2 & b^2 \\ b(a & c) \end{matrix} \quad \begin{matrix} b(a & c) \\ c^2 & c^2 \end{matrix}$

Sum of the diagonal entries of

$A^2 = a^2 + 2b^2 + c^2$

Given $a^2 + 2b^2 + c^2 = 1$, a, b, c $\in$ I

$b = 0$ & $a^2 + c^2 = 1$

Case-1 : $a = 0 \Rightarrow c = \pm 1$ (2-matrices)

Case-2 : $c = 0 \Rightarrow a = \pm 1$ (2-matrices)

Total = 4 matrices

4. In a increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to :

(1) 30 (2) 26 (3) 35 (4) 32

Ans. (**3**)

Sol. a, ar, ar^2, ……

$T_2 \quad T_6 \quad \frac{25}{2} \quad ar(1 \quad r^4) \quad \frac{25}{2}$

$a^2r^2(1 \quad r^4)^2 \quad \frac{625}{4}$ ….(1)

$T_3 . T_5 = 25 \Rightarrow (ar^2)(ar^4) = 25$

$a^2r^6 = 25$ ….(2)

On dividing (1) by (2)

$\frac{(1 \quad r^4)^2}{r^4} \quad \frac{25}{4}$

$4r^8 - 17r^4 + 4 = 0$

$(4r^4 - 1)(r^4 - 4) = 0$

$r^4 \quad \frac{1}{4}, 4 \quad r^4 \quad 4$

(an increasing geometric series)

$a^2r^6 = 25 \Rightarrow (ar^3)^2 = 25$

$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$

$= ar^3 (1 + r^2 + r^4)$

$= 5(1 + 2 + 4) = 35$

5. The value of $\int_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]} dx$, where [x] is the greatest integer $\le$ x, is :

(1) 100(e – 1) (2) 100(1 – e) (3) 100e (4) 100(1 + e)

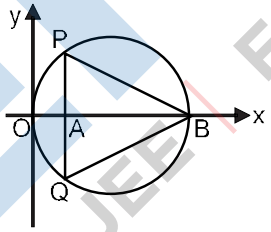
Ans. (1)

Sol. $\int_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]} dx$, period of {x} = 1

$\int_{n=1}^{100} \int_{0}^{1} e^{\{x\}} dx \quad \int_{n=1}^{100} \int_{0}^{1} e^{x} dx$

$\int_{n=1}^{100} (e \quad 1) \quad 100(e \quad 1)$

6. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is :



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$ (3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Ans. (2)

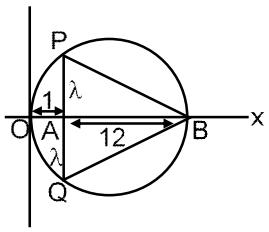
Sol. PA = AQ = λ

OA · AB

= AP · AQ

$\Rightarrow 1.12 = \lambda \cdot \lambda$

$2\sqrt{3}$



Area PQB $\frac{1}{2}$ 2 AB

$\frac{1}{2}$ $4\sqrt{3}$ 12

$24\sqrt{3}$

7. The sum of the infinite series $1 \quad \frac{2}{3} \quad \frac{7}{3^2} \quad \frac{12}{3^3} \quad \frac{17}{3^4} \quad \frac{22}{3^5} \quad .....$ is equal to

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

Ans. (1)

Sol. $S \quad 1 \quad \frac{2}{3} \quad \frac{7}{3^2} \quad \frac{12}{3^3} \quad \frac{17}{3^4} \quad .....$

$\frac{S}{3} \quad \frac{1}{3} \quad \frac{2}{3^2} \quad \frac{7}{3^3} \quad \frac{12}{3^4} \quad .....$

$\frac{2S}{3} \quad 1 \quad \frac{1}{3} \quad \frac{5}{3^2} \quad \frac{5}{3^3} \quad \frac{5}{3^4} \quad .....$ up to infinite terms

$S \quad \frac{13}{4}$

8. The value of $\lim_{h \to 0} 2 \frac{\sqrt{3}\sin \frac{}{6} h \quad \cos \frac{}{6} h}{\sqrt{3}h \ \sqrt{3}\cosh \quad \sinh}$ is

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Ans. (1)

Sol. $L \quad \lim_{h \to 0} 2 \frac{\sqrt{3} \ \frac{1}{2}\cosh \ \frac{\sqrt{3}}{2}\sinh \quad \frac{\sqrt{3}}{3}\cosh \ \frac{\sinh}{2}}{\sqrt{3}h \ \sqrt{3}}$

$L \lim_{h \to 0} \frac{4\sinh}{3h} \qquad L \quad \frac{4}{3}$

9. The maximum value of the term independent of 't' in the expansion of $t^{\frac{1}{5}} \frac{(1 \quad x)^{\frac{1}{10}}}{t}^{10}$ where $x \in (0, 1)$ is :

(1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$ (3) $\frac{2.10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

Ans. (2)

Sol. Term independent of t will be the middle term due to exect same magnitude but opposite sign powers of t in the binomial expression given

So $T_6 \quad {}^{10}C_5(tx^2 5)^5 \frac{(1 \quad x)^{\frac{1}{10}}}{t}^5$

T_6 $f(x)$ $^{10}C_5\ x\sqrt{1\ x}$; for maximum

$f'(x)\ 0\ x\ \frac{2}{3}\ \&\ f''\ \frac{2}{3}\ 0$

So $f(x)_{max.}\ ^{10}C_5\ \frac{2}{3}\ .\frac{1}{\sqrt{3}}$

10. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e \frac{6}{5}}$ hours, then $\frac{k}{\log_e 2}^2$ is equal to :

(1) 4 (2) 8 (3) 2 (4) 16

Ans. (1)

Sol. $\frac{dB}{dt}\ B\ \int_{1000}^{1200}\frac{dB}{B}\ \int_0^2 dt\ \ \frac{1}{2}\ell n\ \frac{6}{5}$

$\int_{1000}^{1200}\frac{dB}{B}\ \frac{1}{2}\ell n\ \frac{6}{5}\ \int_0^T dt\ \ T\ \frac{2\ell n2}{\ell n\ \frac{6}{5}}$

$\Rightarrow k = 2\ \ell n2$

11. If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is :

(1) $\frac{39}{5}$ (2) $\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $\frac{44}{5}$

Ans. (3)

Sol. A(1, 5, 35), B(7, 5, 5), C(1, λ, 7), D(2λ, 1, 2)

$\overline{AB}\ \ 6\hat{i}\ \ 30\hat{k}, \overline{BC}\ \ 6\hat{i}(\ \ 5)\hat{j}\ \ 2\hat{k}, \overline{CD}\ \ (2\ \ 1)\hat{i}\ \ (1\ \ )\hat{j}\ \ 5\hat{k}$

Points are coplanar $0\ \begin{vmatrix} 6 & 0 & 30 \\ 6 & 5 & 2 \\ 2\ \ 1 & 1 & 5 \end{vmatrix}$

$= 6(-5\lambda + 25 - 2 + 2\lambda)$

$- 30(-6 + 6\lambda - (2\lambda^2 - \lambda - 10\lambda + 5))$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 11\lambda - 5 - 6 + 6\lambda)$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 17\lambda - 11)$

$= 6(-3\lambda + 23 + 10\lambda^2 - 85\lambda + 55)$

$= (10\lambda^2 - 88\lambda + 78) = 12(5\lambda^2 - 44\lambda + 39)$

$\Rightarrow 0 = 12(5\lambda^2 - 44\lambda + 39)$

^{1} ^{2} $\frac{44}{5}$

12. If $\frac{\sin^{x} x}{a}$ $\frac{\cos^{1} x}{b}$ $\frac{\tan^{1} y}{c}$;0 x 1, then the value of cos $\frac{c}{a\ b}$ is :

(1) $\frac{1\ y^2}{y\sqrt{y}}$ (2) $1 - y^2$ (3) $\frac{1\ y^2}{1\ y^2}$ (4) $\frac{1\ y^2}{2y}$

Ans. (3)

Sol. $\frac{\sin^{1} x}{r}$ $a, \frac{\cos^{1} x}{r}$ $b, \frac{\tan^{1} y}{r}$ c

So, a b $\frac{}{2r}$

cos $\frac{c}{a\ b}$ cos $\frac{\tan^{1} y}{\frac{}{2r}r}$

= cos(2tan^{-1} y), let tan^{-1} y = θ

= cos(2θ)

$\frac{1\ \tan^2}{1\ \tan^2}$ $\frac{1\ y^2}{1\ y^2}$

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is :

(1) 42 (2) 82 (3) 77 (4) 35

Ans. (3)

Sol. (I) First possibility is 1, 1, 1, 1, 1, 2, 3

required number $\frac{7!}{5!}$ 7 6 42

(II) Second possibility is 1, 1, 1, 1, 2, 2, 2

Required number $\frac{7!}{4!3!}$ $\frac{7\ 6\ 5}{6}$ 35

Total = 42 + 35 = 77

14. Let f be any function defined on R and let it satisfy the condition :

$|f(x)\ \ f(y)|$ $|(x\ \ y)^2|$, (x,y) R

If f(0) = 1, then :

(1) f(x) can take any value in R (2) f(x) < 0, $\forall$ x $\in$ R

(3) f(x) = 0, $\forall$ x $\in$ R (4) f(x) > 0, $\forall$ x $\in$ R

Ans. (4)

Sol. $\left|\frac{f(x) \quad f(y)}{(x \quad y)}\right| \quad |(x \quad y)|$

$x - y = h$ let $\Rightarrow x = y + h$

$\lim\limits_{x \to 0}\left|\frac{f(y \quad h) \quad f(y)}{h}\right| \quad 0$

$\Rightarrow |f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = k$ (constant)

And $f(0) = 1$ given

So, $f(y) = 1 \Rightarrow f(x) = 1$

15. The maximum slope of the curve $y \quad \frac{1}{2}x^4 \quad 5x^3 \quad 18x^2 \quad 19x$ occurs at the point :

(1) (2, 2) (2) (0, 0) (3) (2, 9) (4) $3, \frac{21}{2}$

Ans. (1)

Sol. $\frac{dy}{dx} \quad 2x^3 \quad 15x^2 \quad 36x \quad 19$

Since, slope is maximum so,

$\frac{d^2y}{dx^2} \quad 6x^2 \quad 30x \quad 36 \quad 0$

$\Rightarrow x^2 - 5x + 6 = 0$ | $\frac{d^3y}{dx^3} \quad 12x \quad 30$

$X = 2, 3$ | At $x \quad 2, \frac{d^3y}{dx^3} \quad 0$

at $x = 2$ | So, maxima

$y \quad \frac{1}{2} \quad 16 \quad 5 \quad 8 \quad 18 \quad 4 \quad 19 \quad 2$

$= 8 - 40 + 72 - 38 = 80 - 78 = 2$

Point (2, 2)

16. The intersection of three lines

$x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

(1) Right angled triangle (2) Equilateral triangle

(3) Isosceles triangle (4) None of the above

Ans. (3)

Sol. $L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : x + y = 6$

On solving L_1 and L_2 :

$y = L$ and $x = 1$

L_1 and L_3 :

$x = 2$

$y = 2$

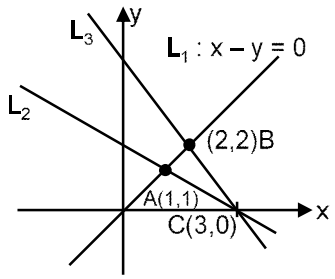
L_2 and L_3 :

$x + y = 3$

$2x + y = 6$

$X = 3$

$Y = 0$



AC $\sqrt{4 \quad 1} \quad \sqrt{5}$

BC $\sqrt{4 \quad 1} \quad \sqrt{5}$

AB $\sqrt{1 \quad 1} \quad \sqrt{2}$

So its an isosceles triangle

17. Consider the three planes

$P_1 : 3x + 15y + 21z = 9,$

$P_2 : x - 3y - z = 5,$ and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true ?

(1) P_1 and P_2 are parallel

(2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel

(4) P_1, P_2 and P_3 all are parallel

Ans. (2)

Sol. $P_1 : x + 5y + 7z = 3,$

$P_2 : x - 3y - z = 5$

$P_3 : x \quad 5y \quad 7z \quad \frac{5}{2}$

So P_1 and P_3 are parallel.

18. The value of $\begin{vmatrix} (a & 1)(a & 2) & a & 2 & 1 \\ (a & 2)(a & 3) & a & 3 & 1 \\ (a & 3)(a & 4) & a & 4 & 1 \end{vmatrix}$ is

(1) $(a + 2)(a + 3)(a + 4)$

(2) -2

(3) (a + 1) (a + 2) (a + 3) (4) 0

Ans. (2)

Sol. $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} (a\ \ 1)(a\ \ 2) & a\ \ 2 & 1 \\ (a\ \ 2)(a\ \ 3\ \ a\ \ 1) & 1 & 0 \\ a^2\ \ 7a\ \ 12\ \ a^2\ \ 3a\ \ 2 & 2 & 0 \end{vmatrix}$$

$$\begin{vmatrix} a^2\ \ 3a\ \ 2 & a\ \ 2 & 1 \\ 2(a\ \ 2) & 1 & 0 \\ 4a\ \ 10 & 2 & 0 \end{vmatrix}$$

$= 4(a + 2) - 4a - 10$

$= 4a + 8 - 4a - 10 = -2$

19. The value of $\int_{/2}^{/2} \frac{\cos^2 x}{1\ \ 3^x} dx$ is

(1) $\frac{}{4}$ (2) 4π (3) $\frac{}{2}$ (4) 2π

Ans. (1)

Sol. $I \int_{/2}^{/2} \frac{\cos^2 x}{1\ \ 3^x} dx$ (using king)

$$I \int_{/2}^{/2} \frac{\cos^2 x}{1\ \ 3^{\ x}} dx \int_{/2}^{/2} \frac{3^x \cos^2 x}{1\ \ 3^x} dx$$

$$2I \int_{/2}^{/2} \frac{(1\ \ 3^x)\cos^2 x}{1\ \ 3^{\ x}} dx$$

$$\int_{/2}^{/2} \cos^2 x dx \quad 2 \int_0^{/2} \cos^2 x dx$$

$$I \int_0^{/2} \cos^2 x dx \quad \frac{}{4}$$

20. Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set :

(1) $S = \{(x, y) \mid x^2 + y^2 = 4\}$ (2) $S = \{(x, y) \mid x^2 + y^2 = 1\}$

(3) $S \quad \{(x,y) \mid x^2 \quad y^2 \quad \sqrt{2}\}$ (4) $S = \{(x, y) \mid x^2 + y^2 = 2\}$

Ans. (4)

Sol. Equivalence class of (1, –1) is a circle with centre at (0, 0) and radius $\sqrt{2}$

$\Rightarrow x^2 + y^2 = 2$

$S = \{(x, y) \mid x^2 + y^2 = 2\}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 \quad a \; x \; \frac{\sqrt{a}}{2}$,a $\quad 0$ is :

Ans. (2)

Sol. $y^2 \quad a \; x \; \frac{\sqrt{a}}{2} \quad ax \; \frac{a^{3/2}}{2}$ ....(1)

$\Rightarrow 2yy' = a$

put in equation (1)

$y^2 \quad (2yy')x \; \frac{(2yy')^{3/2}}{2}$

$(y^2 \quad 2xyy') \; \frac{(2yy')^{3/2}}{2}$

squaring

$(y^2 \quad 2xyy') \; \frac{y^3(y')^3}{2}$

$\therefore$ order = 1

Degree = 3

Degree – order = 3 – 1 = 2

2. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is :

Ans. (11)

Sol. $3 \sin x + 4 \cos x = k + 1$

$k \quad 1 \qquad \sqrt{3^2 \quad 4^2}, \sqrt{3^2 \quad 4^2}$

$\Rightarrow k + 1 \in [-5, 5]$

$\Rightarrow k \in [-6, 4]$

No. of integral values of k = 11

3. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is :

Ans. (1)

Sol. $\log_4(x - 1) = \log_2(x - 3)$

$$\frac{1}{2}\log_2(x \quad 1) \quad \log_e(x \quad 3)$$

$\Rightarrow (x - 1)^{1/2} = \log_2(x - 3)$

$\Rightarrow (x - 1)^{1/2} = x - 3$

$\Rightarrow (x - 1)^{1/2} = \log_2(x - 3)$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x - 2)(x - 5) = 0$

$\Rightarrow x = 2, 5$

But $x \neq 2$ because it is not satisfying the domain of given equation i.e. $\log_2(x - 3) \rightarrow$ its domain $x > 3$

Finally x is 5

$\therefore$ No. of solutions = 1.

4. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

Ans. (3)

Sol. $x^3 - 2x^2 + 2x - 1 = 0$

x = 1 satisfying the equation

$\therefore$ x – 1 is factor of

$x^3 - 2x^2 + 2x - 1$

$= (x - 1)(x^2 - x + 1) = 0$

$$x \quad 1, \frac{1 \quad i\sqrt{3}}{2}, \frac{1 \quad i\sqrt{3}}{2}$$

$x = 1, -\omega^2, -\omega$

sum of 162^{th} power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + (\omega^2)^{324} + (\omega)^{162}$

$= 1 + 1 + 1 = 3$

5. Let m, $n \in N$ and gcd (2, n) = 1. If $30\binom{30}{0} + 29\binom{30}{1} + \ldots\ldots + 2\binom{30}{28} + 1\binom{30}{29} = n.2^m$, then n + m is equal to :

(Here $\binom{n}{k} = {}^nC_k$)

Ans. (45)

Sol. $30({}^{30}C_0) + 29({}^{30}C_1) + \ldots. + 2({}^{30}C_{28}) + 1({}^{30}C_{29})$

$= 30({}^{30}C_{30}) + 29({}^{30}C_{29}) + \ldots. + 2({}^{30}C_2) + 1({}^{30}C_1)$

$\sum_{r=1}^{30} r({}^{30}C_r)$

$\sum_{r=1}^{30} r\,\frac{30}{r}\,({}^{29}C_{r-1})$

$30\sum_{r=1}^{30} {}^{29}C_{r-1}$

$= 30({}^{29}C_0 + {}^{29}C_1 + {}^{29}C_2 + \ldots\ldots. + {}^{29}C_{29})$

$= 30(2^{29}) = 15(2)^{30} = n(2)^m$

$\therefore$ n = 15, m = 30

n + m = 45

6. If y = y(x) is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x, y(0) = 0$; then $\left(1, y\left(\frac{\pi}{6}\right)\right)$ $\left(\frac{\sqrt{3}}{2}, y\left(\frac{\pi}{3}\right)\right)$ $\left(\frac{1}{\sqrt{2}}, y\left(\frac{\pi}{4}\right)\right)$ is equal to :

Ans. (1)

Sol. Put $e^{\sin y} = t$

$e^{\sin y}\cos y\frac{dy}{dx} = \frac{dt}{dx}$

D.E. is $\frac{dt}{dx} + t\cos x = \cos x$

I.F. $= e^{\int \cos x dx} = e^{\sin x}$

$\Rightarrow$ solution is $t.e^{\sin x} = \int \cos x e^{\sin x}$

$\Rightarrow e^{\sin y} e^{\sin x} = e^{\sin x} + c$

$\because$ x = 0, y = 0 $\Rightarrow$ c = 0

$\Rightarrow e^{\sin y} = 1$

$\Rightarrow$ y = 0

$1 \quad y \quad \frac{}{6} \quad \frac{\sqrt{3}}{2} y \quad \frac{}{3} \quad \frac{1}{\sqrt{2}} y \quad \frac{}{4} \quad 1$

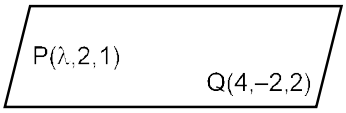
7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\frac{}{11}^{2} \quad \frac{4}{11} \quad 4$ is equal to :

Ans. (8)

Sol.

$A(-2,-21,29)$

$B(-1,-16,33)$



$\overrightarrow{AB} \quad \overrightarrow{PQ} \quad 0$

$\hat{i} \quad 5\hat{j} \quad 6\hat{k} \quad 4 \quad \hat{i} \quad 4\hat{j} \quad \hat{k} \quad 0$

$\Rightarrow 4 - \lambda - 20 - 6 = 0$

$\Rightarrow \lambda = -22$

$\frac{}{11}^{2} \quad \frac{4}{11} \quad 4 \quad 4 \quad 8 \quad 4 \quad 8$

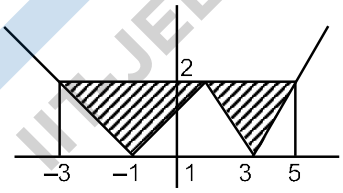
8. The area bounded by the lines $y = ||x - 1| - 2|$ is :

Ans. (8)

Allen Ans by (bonus)

Sol. **Remark :**

Question is incomplete it should be area bounded by $y = |x - 1| - 2|$ and $y = 2$



Area $2 \quad \frac{1}{2}, 4, 2$

9. The value of the integral $\int_{0} |\sin 2x| \, dx$ is :

Ans. (2)

Sol. Put $2x = t \Rightarrow 2dx = dt$

$I \quad \frac{1}{2}^{2}_{0} |\sin t| \, dt$

$$\int_0^{} |\sin t|\, dt$$

$= 2$

10. If $\sqrt{3}(\cos^2 x)$ $\sqrt{3}$ $1 \cos x$ 1, the number of solutions of the given equation when x $0, \frac{}{2}$ is :

Ans. (1)

Sol. $\sqrt{3}(\cos x)^2$ $\sqrt{3}\cos x$ $\cos x$ 1 0

$\sqrt{3}\cos x$ $1\ (\cos x$ $1)$ 0

$\cos x$ 1 or $\cos x$ $\frac{1}{\sqrt{3}}$ (reject)

$\Rightarrow x = 0$ only